Exercise 40

Calculate y'.

$$xe^y = y - 1$$

Solution

Take the derivative of both sides with respect to x.

$$\frac{d}{dx}(xe^y) = \frac{d}{dx}(y-1)$$

$$\left[\frac{d}{dx}(x)\right]e^y + x\left[\frac{d}{dx}(e^y)\right] = \frac{d}{dx}(y) - \frac{d}{dx}(1)$$

$$(1)e^y + x\left[(e^y)\cdot\frac{d}{dx}(y)\right] = \frac{dy}{dx} - 0$$

$$e^y + xe^y\frac{dy}{dx} = \frac{dy}{dx}$$

Solve for dy/dx.

$$e^y = \frac{dy}{dx} - xe^y\frac{dy}{dx}$$

$$e^y = (1 - xe^y)\frac{dy}{dx}$$

Divide both sides by $1 - xe^y$.

$$\frac{dy}{dx} = \frac{e^y}{1 - xe^y}$$

Use the starting equation $xe^y = y - 1$ to simplify the right side.

$$\frac{dy}{dx} = \frac{\left(\frac{y-1}{x}\right)}{1 - (y-1)}$$

$$= \frac{\frac{y-1}{x}}{2 - y} \times \frac{x}{x}$$

$$= \frac{y-1}{x(2-y)}$$